Linear Algebra 2020 Midterm Test

- Write neatly!
- Clearly $\boxed{\text{box}}$ your final answer if there is one.
- There are 10 questions. The first is worth 7, the others worth 5, for a total of 52.
- **Send to mhsiggersknu.ac.kr by 10:30am (KST) THURSDAY, MAY 14.**

1. Answer 'T(rue)' or 'F(alse)'.
   (a) If $AB = AC$ then $B = C$.
   (b) The set of vectors $b = (b_1, b_2, b_3)$ in $\mathbb{R}^3$ with $b_1 - b_2 + 3b_3 = 0$ is a subspace of $\mathbb{R}^3$.
   (c) The matrix $\left[\begin{smallmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 2 \end{smallmatrix}\right]$ has rank 2.
   (d) There are n different $n \times n$ permutation matrices.
   (e) If $(3, 4, 5)$ and $(1, 2, 0)$ are both solutions to $Mx = b$ then M is singular.
   (f) The nullspace of the incidence matrix M of a graph G always has dimension 1.
   (g) If $A^T = -2A$ then the rowspace of A equals the column space of A.
2. Write the following system of equations as a matrix equation, and solve it $2u + v = 5$, $3v - w = 3$, $u + 6w = 7$, $2u - w + z = 12$.
3. Find a 3×3 matrix A with nine distinct entries, but in which row 2 and 3 become zero in elimination. How many solutions does $Ax = b$ have when $b = (1, 10, 100)$ and how many when $b = (0, 0, 0)$.
4. Write each of the following as a single matrix or number.

$$a) \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}^4 \qquad b) \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1}$$

   (Hint: You shouldn't have to do these the long way. Think of what they do as elementary matrices.)
5. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 3 & 2 & 2 \end{bmatrix}$.

6. Solve as two triangular systems (without finding LU) the equation

$$LUx = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}.$$

7. Where A is the matrix Question 5, solve

$$A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}.$$

8. Find bases for the four fundamental subspaces of A where

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 9 & 8 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}.$$

9. What 3×3 matrix represents the linear transformation that rotates the xy-plane by 90° but leaves the z-axis alone?

10. Which of these transformation **are** linear? Where $v = (v_1, v_2)$,

    (a) $T(v) = (v_2, v_2)$.
    (b) $T(v) = (1, v_2)$.
    (c) $T(v) = (-v_2, v_1 + v_2)$.
    (d) $T(v) = (v_1^2, -(v_2^2))$.